Equations and Word Problems

A. Suppose you receive $100 for a graduation present, and you deposit it in a savings account. Then each week thereafter, you add $5 to the account.

1) Write an equation to model this situation.

\[ y = 5x + 100 \]

2) Now, graph this situation:

B. Nick is given $50 to spend on a vacation. He decides to spend $5 a day.

1) Write an equation for this situation.

\[ y = 50 - 5x \]

2) When will Nick have $15 left?

\[ 15 = 50 - 5x \]
\[ -35 = -5x \]
\[ x = 7 \text{ days} \]

3) Graph this situation:
C. An airplane 30,000 feet above the ground begins descending at the rate of 2000 feet per minute. Assume the plane continues at the same rate of descent.

1) Write an equation to model this situation.

\[ y = 30,000 - 2000x \]

2) What will the altitude of the plane be after 5 minutes?

\[ y = 30,000 - 2000(5) \]

\[ 30,000 - 10,000 = 20,000 \text{ feet} \]

D. A cell phone plan costs $30 per month for unlimited calling plus $0.15 per text message.

1) Write an equation to model this situation.

\[ y = 30 + 0.15x \]

2) Make a table of this data:

<table>
<thead>
<tr>
<th>Texts</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>30</td>
<td>31.5</td>
<td>33</td>
<td>34.5</td>
<td>36</td>
</tr>
</tbody>
</table>

E. By noon, a charity had collected 83 shirts. Every hour after that, it collected 20 more shirts. Let \( h \) be the number of hours since noon and \( s \) be the number of shirts.

1) Write an equation to model this situation.

\[ s = 83 + 20h \]

2) After 6 hours, how many shirts had the charity collected?

\[ s = 83 + 20(6) \]

\[ 83 + 120 = 203 \text{ shirts} \]

3) Graph this situation:
Write a function rule that represents each sentence.

1. 5 less than one fourth of \(x\) is \(y\).
   \[\frac{1}{4}x - 5 = y\]

2. 7 more than the quotient of a number \(n\) and 4 is 9.
   \[\frac{n}{4} + 7 = 9\]

3. \(P\) is 9 more than half of \(q\).
   \[P = \frac{1}{2}q + 9\]

4. 8 more than 5 times a number is \(-27\).
   \[5n + 8 = -27\]

5. 1.5 more than the quotient of \(a\) and 4 is \(b\).
   \[\frac{a}{4} + 1.5 = b\]

For Exercises 6–10, write a function rule that represents each situation.

6. The price \(p\) of an ice cream is $3.95 plus $0.85 for each topping \(t\) on the ice cream.
   \[p = 0.85t + 3.95\]

7. A babysitter's earnings \(e\) are a function of the number of hours \(n\) worked at a rate of $7.25 per hour.
   \[e = 7.25n\]

8. The price \(p\) of a club's membership is $30 for an enrollment fee and $12 per week \(w\) to be a member.
   \[p = 12w + 30\]

9. A plumber's fees \(f\) are $75 for a house call and $60 per hour \(h\) for each hour worked.
   \[f = 60h + 75\]

10. A hot dog \(d\) costs $1 more than one-half the cost of a hamburger \(h\).
    \[d = 0.5h + 1\]

11. José is 3 years younger than 3 times his brother's age. Write a rule that represents José's age \(j\) as a function of his brother's age \(b\). How old is José if his brother is 5?
    \[j = 3b - 3; 12\]

12. A taxicab charges $4.25 for the first mile and $1.50 for each additional mile. Write a rule for describing the total rate \(r\) as a function of the total miles \(m\). What is the taxi rate for 12 miles?
    \[r = 1.5(m - 1) + 4.25; \$20.75\]
13. Write a function rule for the area of a rectangle whose length is 4 in. more than its width. What is the area of the rectangle when its width is 8 in.?
   \[ A = (w + 4)w; \ 96 \text{ in}^2 \]

14. Write a function rule for the area of a rectangle with a length 3 ft more than two times its width. What is the area of the rectangle when its width is 4 ft?
   \[ A = (2w + 3)w; \ 44 \text{ ft}^2 \]

15. Write a function rule for the area of a triangle with a base 2 m less than 4 times its height. What is the area of the triangle when its height is 8 m?
   \[ A = \frac{1}{2} (4h - 2)h; \ 120 \text{ m}^2 \]

16. **Reasoning** Write a rule that is an example of a nonlinear function that fits the following description.
   \[ \text{When } b \text{ is 49, } a \text{ is 7, and } a \text{ is a function of } b. \]
   Answers may vary. Sample: \[ a = \sqrt{b} \]

17. **Open-Ended** Describe a real-world situation that represents a nonlinear function.
   Answers may vary. Sample: The height of a soccer ball is a function of the time since it was kicked.

18. **Writing** Explain whether or not the relationship between inches and feet represents a function.
   yes; \[ y = 12x, \text{ where } y \text{ is inches and } x \text{ is feet, is a linear function.} \]

19. **Multiple Representations** Use the table shown at the right.
   a. Graph the ordered pairs on a coordinate plane.
   
   b. Write an equation that can be used to find \( y \) for any \( x \) value.
   \[ y = 2x + 4 \]
   
   c. Is the equation a function? Explain.
   yes; it is a linear function as the points on the graph can be connected with a straight line.
Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Jill earns $45 per hour. Using $p$ for her pay and $h$ for the hours she works, what function rule represents the situation? $B$
   
   A. $h = 45p$
   B. $p = 45h$
   C. $h = p + 45$
   D. $p = h + 45$

2. What is a function rule for the perimeter $P$ of a building with a rectangular base if the width $w$ is two times the length $l$? $H$
   
   F. $P = 2l$
   G. $P = 2w$
   H. $P = 6l$
   I. $P = 6w$

3. Which function rule can be used to represent the area of a triangle with a base $b$ 8 in. longer than twice the height $h$ in terms of the height? $C$
   
   A. $A = \frac{1}{2}bh$
   B. $A = \frac{1}{2}h(h + 8)$
   C. $A = h^2 + 4h$
   D. $A = \frac{1}{2}(2h)(h + 8)$

4. Which equation represents the sentence “$d$ is 17 less than the quotient of $n$ and 4”? $F$
   
   F. $d = \frac{n}{4} - 17$
   G. $d = \frac{n}{4} + 17$
   H. $d = 4n - 17$
   I. $d - 17 = \frac{n}{4}$

5. The function rule for the profit a company expects to earn is $P = 1500m + 2700$, where $P$ represents profit and $m$ represents the number of months the company has been in business. How much profit should the company earn after 12 months in business? $D$
   
   A. $15,700$
   B. $17,700$
   C. $18,000$
   D. $20,700$

Extended Response

6. A plane was flying at an altitude of 30,000 feet when it began the descent toward the airport. The airplane descends at a rate of 850 feet per minute.
   
   a. What is the function rule that describes this situation? $A = 30,000 - 850m$
   
   b. What is the altitude of the plane after it has descended for 8 minutes? Show your work. 23,200 ft
   
   c. Use the function in part a to determine how long it takes for the plane to land if it descends at a continuous rate.

   [1] One or two parts answered correctly.
   [0] No parts answered correctly.
Moving Straight Ahead Word Problems

A tree is 34 inches tall and grows 3 inches each week.

a. Write an equation that represents the tree's height (h) over time (t).
   \[ h = 34 + 3t \]

b. What is the slope of the line your equation represents? What does the slope tell you about this situation?
   \[ m = 3; \text{ it means the tree grows 3 in/\text{day}} \]

c. What is the y-intercept of the line? What does the y-intercept tell you about the situation?
   \[ b = 34; \text{ it means the tree's starting height is 34 in} \]

d. How tall will the tree be in 9 weeks?
   \[ h = 34 + 3(9) = 34 + 27 = 61 \text{ inches} \]

e. If the tree is 94 inches tall, for how many weeks has it been growing?
   \[
   \begin{align*}
   94 &= 34 + 3t \\
   3t &= 94 - 34 \\
   t &= \frac{60}{3} = 20 \text{ weeks}
   \end{align*}
   \]

2) Maria rented a jet ski while she was on vacation this past summer. She paid a rental fee of $75 plus $22.50 each day to rent the jet ski.

a. Write an equation to find Maria's total charge, C, for renting the jet ski for d days.
   \[ C = 75 + 22.50d \]

b. What is the slope of the line your equation represents? What does the slope tell you about this situation?
   \[ m = 22.50; \text{ it means it costs $22.50 each day} \]

c. What is the y-intercept of the line? What does the y-intercept tell you about the situation?
   \[ b = 75; \text{ it means the upfront cost is $75} \]

d. How much will it cost Maria if she rents the jet ski for 4 days?
   \[ C = 75 + 22.50(4) = 75 + 90 = \$165 \]

e. If Maria's total charge was $232.50, how many days did she rent the jet ski?
   \[
   \begin{align*}
   232.50 &= 75 + 22.50d \\
   157.50 &= 22.50d \\
   d &= \frac{157.50}{22.50} = 7 \text{ days}
   \end{align*}
   \]

3) The temperature in Frostburg is 12°F and dropping at the rate of 2.5 degrees per hour.

a. Write an equation to find the temperature in Frostburg, T, for h number of hours.
   \[ T = 12 - 2.5h \]

b. What is the slope of the line your equation represents? What does the slope tell you about this situation?
   \[ m = -2.5; \text{ it means the temperature is dropping 2.5 degrees/hr} \]

c. What is the y-intercept of the line? What does the y-intercept tell you about the situation?
   \[ b = 12; \text{ it tells us what the temperature is at the beginning} \]

d. What will the temperature be in Frostburg in 5 hours?
   \[ T = 12 - 2.5(5) = 12 - 12.5 = -0.5°F \]

e. How many hours will it be until the temperature is -10.5°F?
   \[
   \begin{align*}
   -10.5 &= 12 - 2.5h \\
   -23 &= -2.5h \\
   h &= \frac{-23}{-2.5} = 9 \text{ hours}
   \end{align*}
   \]
4) The FBLA Club is having some posters printed. The printer charges $180 plus $2.25 per poster.

a. Write an equation to find the FBLA Club's total charge, $C$, for printing $p$ posters.

$$C = 180 + 2.25p$$

b. What is the slope of the line your equation represents? What does the slope tell you about this situation?

$m = 2.25$; it means it costs $2.25$ per poster.

c. What is the y-intercept of the line? What does the y-intercept tell you about the situation?

$b = 180$; it means the upfront cost is $180.

d. How much will it cost the FBLA club if they purchase 300 posters?

$$C = 180 + 2.25(300) = 180 + 675 = \$855$$

e. How many posters can be printed for $1530?

$$\frac{1530 - 180}{2.25} = 600$$

5) The school drama club is performing a play at the community theater. Props cost $250 and the theater is charging the drama club $1.25 for each ticket sold. Customers pay $4 for each ticket.

a. Write an equation to represent the expenses of the drama club.

$$E = 250 + 1.25t$$

b. Write an equation to represent the income of the drama club.

$$I = 4t$$

c. What is the drama club's profit or loss if 213 tickets are sold?

$$E = 250 + 1.25(213); E = 614.25$$

$$I = 4(213)$$

$$I = 852$$

They make a profit of $337.75.

d. If the total ticket sales are $780, how many people attended the play?

Income: $780 = 4t$

$$t = 195$$

Income: $780 = 4t$

$$780 = 4t$$

$$t = 195$$

337.75

e. What is the cost of putting on the play for the number of people you found in part (d)?

$$E = 250 + 1.25(195)$$

$$E = 493.75$$

f. How many tickets does the drama club need to sell to break even?

$$E = I$$

$$250 + 1.25t = 4t$$

$$1.25t = 1.25t$$

$$250 = 2.75t$$

$$90.9 = t$$

At about 91 tickets, they will break even.